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Instantons in $c = 0$ CSFT

Robert de Mello Koch[bc], Antal Jevicki[a] and João P. Rodrigues[b]

[a] *Department of Physics, Brown University*
Providence, RI 02912, U.S.A.
[b] *Department of Physics and Centre for Theoretical Physics*
University of the Witwatersrand
Wits, 2050, South Africa
[c] *Stellenbosch Institute for Advanced Studies*
Stellenbosch, South Africa
E-mail: robert@neo.phys.wits.ac.za, antal@het.brown.edu,
rodriguesj@physics.wits.ac.za

Abstract: Following the recent work of [17] we discuss the emergence of D-brane instanton solutions in $c = 0$ noncritical string theory. Our emphasis is on finding the D-instanton effects in a field theoretic setting. Using the framework of single matrix collective field theory (CSFT) we exhibit the appearance of such solutions. Some subtle issues regarding the form of the field theory equations, the comparison with string equations and the importance of a finite N exclusion principle are also discussed.

Keywords: String Field Theory, M(atrix) Theories, D-branes.

Contents

1. Introduction

Noncritical $c \leq 1$ string theory has always been a useful laboratory for studying both perturbative and nonperturbative phenomena in string theory [1−6]. Through the large-N matrix duality nonlinear (string) equations were derived for these theories in the early 1990's and the first insight into nonperturbative D-brane phenomena was obtained.

Recently the subject has been studied with renewed vigour [7−22]. A highly nontrivial calculation was accomplished in ref [17] where the D-instanton contribution to the $c = 0$ string partition function was evaluated. The explicit numerical result

$$\frac{i}{8\sqrt{\pi}\,3^{3/4}\,t^{5/8}}\,e^{-\frac{8\sqrt{3}}{5g_s}\,t^{5/4}} \tag{1.1}$$

exhibits the action of the instanton (in the exponential) and the prefactor specifying the overall weight (chemical potential) of the instanton contribution. The classical action of the instanton has been known for some time; it can be evaluated in particular from the above mentioned string or loop equations [23, 24]. It can also be computed as the disk amplitude in conformal Liouville theory. The evaluation of the overall coefficient accomplished in ref [17] required the original, matrix integral representation of $c = 0$ string theory. As was emphasized in ref [17] this overall coefficient is not yet computable in conformal (Liouville) theory. Furthermore attempts to obtain this effect (and the instanton solution itself) in the field theoretic formalism of loop equations or KdV type string equations have exhibited clear difficulties.

In the present work, we consider the issue of obtaining the D-instanton effect in field theoretic terms. We use the framework of collective field theory [25] that was successful in various studies of $c = 1$ noncritical string theory. In the present case ($c = 0$) it is convenient to use a stochastic framework, which stabilizes the theory in a manner similar to the Marinari-Parisi [31] framework, combined with density function collective field theory. Generally, we will call this field theoretic representation CSFT. It is similar in form the stochastic approach to loop space string field theory established in refs [23, 24], but with some subtle differences which we will exhibit in the text. Some of these differences do play a role in the question of deriving the instanton effect in this version of field theory.

The content of our paper goes as follows. In section 2 we discuss some basic facts involved in the collective integral representation. In particular the role of the jacobian is emphasized. We explain that the presence of this jacobian plays a crucial role in supplying the correct weight in the instanton sector. In section 3 we describe the stochastic (Fokker-Planck) version of collective field theory. We comment on the similarities (and differences) in comparison to loop space field theory and discuss how the string equations emerge in the scaling limit. In section 4 we exhibit and solve the equations characterizing the single eigenvalue instanton. Section 5 is reserved for conclusions.

2. Basics

In this section, we begin by outlining some (well known) basic elements of collective field theory. One starts with the $c = 0$ partition function in the eigenvalue representation

$$Z_N = \int \prod_{i=1}^{N} dx_i \Delta^2 \, e^{-\sum_i V(x_i)} \,. \tag{2.1}$$

The field theoretic representation is achieved by changing to the collective field (density of eigenvalues)

$$\rho(x) = \sum_{i=1}^{N} \delta(x - x_i) \,. \tag{2.2}$$

The potential with the van der Monde measure gives rise to the action

$$S(\rho) = -\frac{1}{2} \int dx \, dy \, \rho(x) \ln(x-y)^2 \rho(y) + \int dx \, \rho(x) V(x) \,. \tag{2.3}$$

The partition function is now

$$Z_N = \int \prod_x d\rho(x) \, J_N(\rho) e^{-S(\rho)} \tag{2.4}$$

with a nontrivial jacobian $J_N(\rho)$ arising through the change of variable

$$J_N(\rho) = \int \prod_{i=1}^{N} dx_i \prod_x \delta\left(\rho(x) - \sum_{i=1}^{N} \delta(x - x_i)\right) . \tag{2.5}$$

It is this jacobian (which is usually ignored) that will play some role in the field theoretic treatment of the eigenvalue instanton.

The jacobian is nontrivial both in its nonpolynomial functional dependence on the collective density and its nontrivial scaling properties (with respect to N). An explicit (power series) expansion for the jacobian as function of $\rho(x)$ can be generated as follows. Using a Lagrange multiplier $\psi(x)$ one has

$$J_N(\rho) = \int [d\psi]\, e^{i\int \psi(x)\rho(x)dx} \left[\frac{1}{L}\int e^{-i\psi(x')}dx'\right]^N . \tag{2.6}$$

In the limit $N \to \infty$, $L \to \infty : \rho_0 = N/L$ we have

$$\lim_{N,L\to\infty} \left(\frac{1}{L}\int dx\, e^{-i\psi(x)}\right)^N = e^{\rho_0 \int dx \left[e^{i\psi(x)}\right]_{\mathrm{ir}}} \tag{2.7}$$

where ir denotes the irreducible part. This representation implies that the collective field theory can be written in terms of the two coupled field $\rho(x)$ and $\psi(x)$ with the action

$$S[\rho,\psi] = \int dx \left(i\psi(x)\rho(x) + \frac{N}{L}\left[e^{-i\psi(x)}\right]\right) + S(\rho) \tag{2.8}$$

and with a trivial measure i.e. no jacobian. Since $\rho(x)$ appears quadratically in the above, it can also be eliminated resulting in a lagrangian for $\psi(x)$ only. We will not use this "dual" representation in this work and will not pursue it further.

The jacobian enforces several nontrivial features contained in the above transformation. First one has an infinite chain of constraints. These can be described in terms of the moments $\rho_n = \int dx\, x^n \rho(x)$ as follows: introduce the Schur polynomials, ref [26]

$$P_n(\rho_1, \rho_2, \rho_3, \ldots) . \tag{2.9}$$

The constraints can be written as

$$P_{N+n}(\rho_1, \rho_2, \rho_3 \ldots) = 0. \tag{2.10}$$

They imply the fact that the variables ρ_{N+n} are dependent on $\rho_1, \rho_2, \ldots \rho_{N-1}$. We see that the jacobian enforces the exclusion principle which manifests itself as a cut off at $n = N$. The second property which can be seen to follow from the jacobian is a recursion involving $N \to N-1$. This will be of crucial importance for the instantons.

Let us then consider the one-instanton sector obtained by separating a single eigenvalue. Write

$$\rho(x) = \rho'(x) + \delta(x-y) \tag{2.11}$$

where for $\rho'(x) : \int dx \rho'(x) = N - 1$. In this case

$$\int [d\rho]\, J_N(\rho+\delta) = N \int dy \int [d\rho']\, J_{N-1}(\rho') . \tag{2.12}$$

Consequently

$$Z_N = Z_N^{(0)} + N \int dy \int [d\rho']\, J_{N-1}(\rho')\, e^{-S(\rho'+\delta)} + \cdots \tag{2.13}$$

where $Z_N^{(0)}$ denotes the no instanton sector partition function. Here we see that the recursion property of the jacobian supplies a crucial factor of N in the weight of the 1 instanton contribution. The integral over the location of the instanton y is a standard field theory collective coordinate integration.

The rest of the calculation now proceeds by evaluating the 1-instanton sector functional integral through the stationary point method. The action becomes

$$S(\rho+\delta) = -\frac{1}{2}\int \rho'(x)\ln(x-x')^2\rho'(x')\,dx\,dx' + \int V(x)\rho'(x)\,dx - 2\int \rho'(x)\ln(y-x)\,dx \tag{2.14}$$

where

$$\int dx\rho'(x) = N-1 = N'\,. \tag{2.15}$$

Rescaling ρ' and x we have

$$\frac{S}{N'^2} = -\frac{1}{2}\int \rho(x)\ln(x-x')^2\rho(x')\,dx\,dx' + \int V(x)\rho\,dx - \frac{2}{N'}\int \rho\ln(y-x)\,dx \tag{2.16}$$

and for the rest of the discussion that follows we will have that $\rho(x)$ is normalized to $1 : \int dx\rho(x) = 1$. The equations of motion following from this (shifted) action are

$$2\int dx'\rho(x')\log(x-x') = V(x) - \frac{2}{N'}\ln(y-x)\,. \tag{2.17}$$

The derivative with respect to x of this equation takes the form of a BIPZ one matrix integral equation [27]

$$2⨍ dx'\rho(x')\frac{1}{x-x'} = V'(x) + \frac{2}{N'}\frac{1}{y-x} \equiv V'(x) + \frac{1}{N'}\Delta V'\,. \tag{2.18}$$

This equation can be exactly solved using the standard methods as will be described in appendix A. In this section, we will adopt a field theoretic approach allowing a construction of $\rho(x)$ to first order in $\frac{1}{N'}$ since we see that the eigenvalue term in the equation represents a source with a coupling proportional to 1/N . Therefore one can expand

$$\rho(x) = \rho_0(x) + \frac{1}{N'}\rho_1(x) + O\left(\frac{1}{N'^2}\right) \tag{2.19}$$

where the leading order $\rho_0(x)$ represents a solution to the equation

$$⨍ \frac{\rho_0(x')}{x-x'}dx' = \frac{1}{2}V'(x)\,. \tag{2.20}$$

The correction $\rho_1(x)$ can be expressed in terms of the density Greens function [28] which reads

$$K_2(x,x') = \frac{1}{2\pi^2}\frac{1}{(x-x')^2}\frac{xx' - \frac{1}{2}(a_0+b_0)(x+x') + a_0b_0}{\sqrt{(x-a_0)(x-b_0)(x'-a_0)(x'-b_0)}} \tag{2.21}$$

as

$$\rho_1(x) = \int dx' K_2(x,x')\Delta V(x') = -2\int dx' K_2(x,x')\log(y-x')\,. \tag{2.22}$$

To perform the above integral, it is useful to write the Greens function as

$$K_2 = \frac{1}{2\pi^2}\frac{1}{\sqrt{a_0-x}\,\sqrt{x-b_0}}\frac{\partial}{\partial x'}\frac{\partial}{\partial x}\left(\sqrt{a_0-x'}\sqrt{x'-b_0}\log|x-x'|\right)\,. \tag{2.23}$$

After an integration by parts, we obtain

$$\rho_1(x) = \frac{1}{\sqrt{a_0 - x}\,\sqrt{x - b_0}}\frac{1}{y - x}\int_{b_0}^{a_0}\frac{dx'}{\pi^2}\sqrt{a_0 - x'}\sqrt{x' - b_0}\left[\frac{1}{x - x'} - \frac{1}{y - x'}\right]. \tag{2.24}$$

Next consider the computation of integrals of the form

$$I = \int_{b_0}^{a_0} dx'\sqrt{a_0 - x'}\,\sqrt{x' - b_0}\frac{1}{x - x'}. \tag{2.25}$$

After a shift in the integration variable, this integral reduces to computing the Hilbert transform

$$\frac{1}{\pi}\int_{-a}^{a}\frac{\sqrt{a^2 - x^2}}{\lambda - x}dx = \lambda - \sqrt{\lambda^2 - a^2}, \tag{2.26}$$

for $a < \lambda$ and

$$\frac{1}{\pi}\int_{-a}^{a}\frac{\sqrt{a^2 - x^2}}{\lambda - x}dx = \lambda, \tag{2.27}$$

for $a > \lambda$. Thus,

$$\rho_1(x) = -\frac{1}{\pi}\frac{1}{y - x}\frac{\sqrt{y - a_0}\,\sqrt{y - b_0}}{\sqrt{a_0 - x}\,\sqrt{x - b_0}} + \frac{1}{\pi}\frac{1}{\sqrt{a_0 - x}\,\sqrt{x - b_0}}. \tag{2.28}$$

Regarding this expression we note that the coefficient of $\frac{1}{y-x}$ can be seen to match with the exact result given in appendix A.

It should be noted that although in this paper we concentrate on the $c = 0$ criticality, this result is universal and independent of the potential, as it was obtained from the universal correlator of [28] together with the logarithmic source associated with the instanton eigenvalue.

We are now in a position to evaluate the partition function in the one instanton sector. We have already seen that in this case the partition function takes the form

$$Z_N^{(1)} = N\int dy\int[d\rho']\,J_{N-1}(\rho')\,e^{-S(\rho'+\delta)}. \tag{2.29}$$

Evaluating this through the stationary point method gives

$$\begin{aligned} Z_N^{(1)} &= N\int_{\text{out}} dy\, e^{-S^1(\rho_0 + \frac{1}{N'}\rho_1)} \\ &= Z_{N-1}^{(0)}N\int_{\text{out}} dy e^{2N'\int\rho_0(x)\ln(y-x)dx + 2\int\rho_1(x)\ln(y-x)dx}. \end{aligned} \tag{2.30}$$

where we have identified the no instanton partition function

$$Z_{N-1}^{(0)} = e^{-N'^2 S(\rho_0)} \tag{2.31}$$

for the $N - 1 \times N - 1$ matrix model. The label "out" for the y integration region indicates that we are integrating out of the region of ρ_0's support, that is $y > a_0$ and $y < b_0$. The $O(1)$ piece of the integrand

$$e^{2N'\int\rho_0(x)\ln(y-x)dx + 2\int\rho_1(x)\ln(y-x)dx} \tag{2.32}$$

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is independent of the potential. Evaluating this contribution we obtain

$$2\int \rho_1(x)\ln(y-x)dx = 4\int \log(y-x')K_2(x',x)\log(y-x)$$
$$= 2\log\left[1+\frac{y-\frac{a_0+b_0}{2}}{\sqrt{y-a_0}\sqrt{y-b_0}}\right] - 2\log 2\,, \tag{2.33}$$

where we see agreement with [17]. Continuing, the complete contribution of the 1-instanton normalized with respect to the no instanton (vacuum) partition function is given by

$$\mu = \frac{Z_N^{(1)}}{Z_N^{(0)}} = \frac{N}{4}\int_{\text{out}} dy \frac{Z_{N-1}^{(0)}}{Z_N^{(0)}}\left[1+\frac{y-\frac{a_0+b_0}{2}}{\sqrt{y-a_0}\sqrt{y-b_0}}\right]^2 e^{-NV(y)-2N\int \rho_0(x)\ln(y-x)dx}\,. \tag{2.34}$$

The remaining integral over y is evaluated through the stationary point method. Here for the ratio of partition functions we have

$$\frac{Z_{N-1}^{(0)}}{Z_N^{(0)}} = \frac{e^{-N'^2 S(\rho_0)}}{e^{-N^2 S(\rho_0)}} = e^{(2N-1)S(\rho_0)}\,. \tag{2.35}$$

Notice that it is the leading order free energy that determines this ratio. The equation (2.34) for μ above represents a general expression for a single eigenvalue contribution, written in terms of a generic potential $V(x)$ and can be used to take the scaling limit. As emphasized by ref [17] the presence of the large-N factor N is crucial for obtaining the correct weight (chemical potential) of the instanton. In our derivation the factor of N is seen to be associated with the field theoretic measure contained in the jacobian $J(\rho)$. In turn any field theory not containing such a measure will not be able to provide a correct (i.e. in agreement with a matrix model) prediction of the instanton effect.

3. Fokker-Planck collective field theory

The most natural scheme for deriving elements of string field theory from matrix models is through stochastic quantization. Stochastic (and also the Marinari-Parisi) formulation of the one matrix integral provides a stabilization of the model and was as such originally introduced in [29, 30].

For the single matrix problem, for a general action $S = \operatorname{Tr} V(M)$, one has the Fokker-Planck hamiltonian

$$H_{FP} = -\frac{1}{2}\operatorname{Tr}\left(\frac{\partial}{\partial M} - \frac{\partial S}{\partial M}\right)\frac{\partial}{\partial M} \tag{3.1}$$

whose hermitean form is

$$\hat{H} = -\frac{1}{2}\left[\frac{\partial}{\partial M} - \frac{1}{2}\frac{\partial S}{\partial M}\right]\left[\frac{\partial}{\partial M} + \frac{1}{2}\frac{\partial S}{\partial M}\right]. \tag{3.2}$$

This equals

$$\hat{H} = -\frac{1}{2}\left(\frac{\partial^2}{\partial M^2}\right) + \frac{1}{4}\left[S^{(1)}(M), P\right] + \frac{1}{8}\operatorname{Tr}\left(S^{(1)}(M)^2\right) \tag{3.3}$$

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with $P = \partial/\partial M$ and $S^{(1)}(M) = \partial S/\partial M$. A similar form is also associated with the Marinari-Parisi supersymmetric one dimensional matrix model [31]. A heuristic way to obtain a collective field representation for this theory is by the replacement

$$M \to \lambda_i \tag{3.4}$$

$$P \to -\frac{(1-\delta_{ij})}{(\lambda_i - \lambda_j)} \tag{3.5}$$

so that

$$\mathrm{Tr}\left[S^{(1)}(M), P\right] = -\sum_{i \neq j} \frac{v^{(1)}(\lambda_i) - v^{(1)}(\lambda_j)}{\lambda_i - \lambda_j} . \tag{3.6}$$

In terms of the density variable

$$\phi(x) = \sum_i \delta(x - \lambda_i) \tag{3.7}$$

one has

$$\begin{aligned} H_{coll} = & \int \frac{1}{2}\left[\phi \Pi_{,x}^2 + \frac{\pi^2}{3}\phi^3 + \frac{1}{8}\left(v^{(1)}(x)\right)^2 \phi(x)\right] dx \\ & - \frac{1}{4}\int dx dy \, \frac{v^{(1)}(x) - v^{(1)}(y)}{x-y} \, \phi(x)\phi(y). \end{aligned} \tag{3.8}$$

This field theory was introduced first in connection with a *supersymmetric* generalization of ordinary collective field theory [32] and was studied in more detail in [33]. A more rigorous derivation of the extra terms will be given latter. This hermitian hamiltonian can be written as

$$H = \int \frac{1}{2}\phi \Pi_{,x}^2 \, dx + \int \frac{1}{2}\phi(x)\left(⨍ \frac{1}{x-y}\,\phi(y)\,dy - \frac{v'(x)}{2}\right)^2 dx \, . \tag{3.9}$$

Its static stationary point equation is

$$⨍ \frac{\phi(y)}{x-y} dy = \frac{v'(x)}{2} \tag{3.10}$$

which is the BIPZ one matrix integral equation. We mention that the above, nonlocal form for the collective hamiltonian is also used in studies of extremal, BPS soliton type solutions ref [34].

Because of later relevance we comment at this point that the structure involved in the above hamiltonian is also involved in the Schwinger-Dyson approach which is based on using loop variables $\phi_n = \mathrm{Tr}(M^n)\ n > 0$ or $\phi(\ell) = \mathrm{Tr}(e^{-\ell M})\,\ell \geq 0$. Equivalently one has the resolvent

$$\Phi(z) = \mathrm{Tr}\left(\frac{1}{z-M}\right) . \tag{3.11}$$

In comparison the collective density field equals

$$\phi(x) = \mathrm{Tr}\,(\delta(x-M)) = \int \frac{dk}{2\pi}\, e^{ikx}\ \mathrm{Tr}\left(e^{-ikM}\right) \tag{3.12}$$

with $k \lesseqgtr 0$. Consequently we can think of this as *also* adding negative loops $\phi(-l) = \mathrm{Tr}(e^{lM})$, then after an analytic continuation ($l \to ik$) one obtains

$$\phi(k) = \mathrm{Tr}(e^{ikM}) \qquad -\infty < k < +\infty. \tag{3.13}$$

We now proceed to the more detailed discussion of how a hermitian collective field hamiltonian is derived. One has first that

$$H = -\int_{-\infty}^{\infty} dk \left[\int_{-\infty}^{\infty} dk'\,\Omega(k,k')\Pi_{k'} + \omega(k) - \Omega(S,k)\right] k\Pi(k) \tag{3.14}$$

with

$$\Pi(k) = \frac{\delta}{\delta\phi(k)} \tag{3.15}$$

$$\Omega(k,k';\phi) = \frac{\partial\phi_k}{\partial M}\frac{\partial\phi_{k'}}{\partial M} = -kk'\phi(k+k') \tag{3.16}$$

$$\omega(k) = \mathrm{Tr}\,\partial_M^2\phi(k) = -k^2\int_0^1 d\alpha\phi(k\alpha)\phi\left(k(1-\alpha)\right) \tag{3.17}$$

and

$$\Omega(S,k) = \mathrm{Tr}\,\frac{\partial S}{\partial M}\frac{\partial\phi_{(k)}}{\partial M} = \int_{-\infty}^{\infty}\Omega(k,k')\,\frac{\delta S}{\delta\phi_{k'}}\,dk'. \tag{3.18}$$

For comparison, the loop space field theory would also involve the operator

$$O_\ell = \phi_{\ell+\ell'}\ell'\Pi_{\ell'} + \phi_{\ell-\ell'}\phi_{\ell'} - \phi_{\ell+\ell'}\ell'\frac{\delta V}{\delta\phi_{\ell'}}. \tag{3.19}$$

It then involves a shift

$$\Phi(z) = \frac{1}{2}V^{(1)}(z) + \varphi(z) \tag{3.20}$$

after which φ is taken to continuum limit. In principle this can be problematic. Recall that $\Phi(z) = \sum_{n\geq 0} z^{-n-1}\phi_n$ while $V(z) = \sum t_n z^n$ (positive powers). Consequently one is attempting to cancel positive powers through a shift of a variable containing only negative ones. Similarly in loop notation the quadratic term

$$\int_0^\ell \ell'\phi_{\ell-\ell'}\phi_{\ell'} \tag{3.21}$$

involves loops of length $\leq \ell$ while the linear term

$$\phi_{\ell+\ell'}\,\ell'\,V_{\ell'}^{(1)} \tag{3.22}$$

contains a loop of length $> \ell$.

Some of these issues are not there in the density collective representation which involves both positive and negative loops. The density variable being real and since the F-P hamiltonian was hermitian the next (important) step in the collective formalism corresponds to a similarity transformation which provides a manifestly hermitian representation for the hamiltonian

$$H = \frac{1}{2}\int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk'\,\tilde{O}_k\,\Omega^{-1}(k,k')\,\tilde{O}_{k'}^{+} \tag{3.23}$$

with

$$O_k = \Omega(k,k')k'\Pi_{k'} + \frac{1}{2}\left(\omega(k) - \Omega(S,k)\right) \tag{3.24}$$

$$O_k^+ = -k\Pi_{k'}\Omega(k,k') + \frac{1}{2}\left(\omega(k) - \Omega(S,k)\right). \tag{3.25}$$

Apart from ordering terms this gives

$$H = \int dx \frac{1}{2}\phi\, \Pi_{,x}^2 + V_{\text{eff}} \tag{3.26}$$

with

$$V_{\text{eff}} = \frac{1}{8}\left[\omega(k) - \Omega(s,k)\right]\Omega^{-1}(k,k')\left[\omega(k') - \Omega(S,k')\right]. \tag{3.27}$$

Using

$$\Omega(x,y) = \partial_x\partial_y\left(\delta(x-y)\phi(x)\right) \tag{3.28}$$

$$\omega(x) = 2\partial_x\left(\phi \fint \frac{1}{x-y}\phi(y)dy\right) \tag{3.29}$$

we obtain the total hamiltonian written as

$$H = \frac{1}{2}\int dx\left[\phi\Pi_{,x}^2 + \phi(x)\left(\fint \frac{\phi(x')}{x-x'}dx' - \frac{1}{2}v'(x)\right)^2\right]. \tag{3.30}$$

The effective potential term in this hamiltonian involves the Hilbert transform and as such looks non-local. Using a certain cubic identity, it can be also be written in the manifestly local form

$$H = \int \frac{1}{2}\left[\left(\phi\Pi_{,x}^2 + \frac{\pi^2}{3}\phi^3\right) + \frac{1}{8}\left(v'(x)\right)^2\phi(x)\right]dx - \frac{1}{4}\int dx\,dy\,\frac{v'(x)-v'(y)}{x-y}\,\phi(x)\phi(y). \tag{3.31}$$

Regarding the two different versions of the field theory hamiltonian we note that their equivalence is based on a formal identity. In particular, this might only be strictly true when there are no divergences (in the energy). In general the term involving the original Hilbert transform and the local cubic term do not necessarily regularize the divergences in the same way [35]. This issue, we believe, is similar to the issue of surface terms in a typical theory of gravity. As is well known, in any gauge fixed version of gravity, surface terms have to be carefully adjusted. In the above, this observation is of particular relevance for the appearance (or nonappearance) of instanton solutions. As we will see, the separation of single eigenvalue instantons is possible in the first version of the theory but very questionable in the second.

The second local version where the interaction is given by a simple local cubic term corresponds to the dynamical Fermi surface picture which provided great insight in studies of the c=1 string theory ref [36]. This Fermi surface is also very useful for making contact with the scaling string equations. Take for example the potential relevant for the simplest k=2 case:

$$v(x) = \frac{x^2}{2} - \frac{g}{3}x^3, \tag{3.32}$$

then

$$v'(x) = x - gx^2 \tag{3.33}$$

and the extra term reads

$$-\frac{1}{4}\int dx \int dy[1 - g(x-y)]\phi(x)\phi(y) = -\frac{1}{4}\left[\left(\int dx\phi\right) - 2g\left(\int \phi dy\right)\left(\int dx x\phi(x)\right)\right]. \tag{3.34}$$

Normalizing $\int \phi = 1$ we have

$$V_{\text{eff}} = \frac{1}{2}\left(\frac{\pi^2}{3}\phi^3(x) + \left[gx + \frac{1}{4}(x - gx^2)^2\right] + c_0\right), \tag{3.35}$$

with the static equation

$$\pi^2\phi^2(x) + \left(gx - c + \frac{1}{4}(x - gx^2)^2\right) = 0\,. \tag{3.36}$$

The double scaling limit is taken as before

$$x = x^* - \gamma a y \tag{3.37}$$

$$g = g^*\left(1 - \frac{\sqrt{3}}{16}\gamma^2 a^2 t\right) \tag{3.38}$$

with

$$g^* = \frac{3^{1/4}}{6}\,, \qquad x^* = (\sqrt{3}+1)3^{1/4} \tag{3.39}$$

to give (to obtain this equation we have set $\gamma = 2^{2/3}3^{5/12}$)

$$(\pi\tilde{\phi})^2 - \left(y^3 - \frac{3}{4}\Lambda y + \frac{1}{4}\Lambda^{3/2}\right) = 0\,. \tag{3.40}$$

Continuing to the hamiltonian we have that the time of flight

$$T = -\int \frac{dy}{\tilde{\phi}(y)} \approx a^{2/3}a\left(2\sqrt{\frac{2\omega}{3}}\right)\int \frac{dx}{\pi\phi_0(x)} \tag{3.41}$$

is finite

$$T = \lim_{a\to 0} a^{1/2}\frac{1}{\sqrt{a}} = O(1), \tag{3.42}$$

and that the hamiltonian becomes

$$H = \epsilon^{1/2}\int dy\left(\frac{1}{2\kappa}(\partial\eta)\pi^2 + \frac{1}{6\kappa}(\partial\eta)^3 + \frac{1}{2}\phi_0(y)\left(\pi^2 + (\partial_y\eta)^2\right)\right). \tag{3.43}$$

Consequently after rescaling we have a finite hamiltonian which takes the form of a dynamical Fermi surface theory:

$$\mathcal{H} = \int \frac{dy}{2\pi}\int_{\alpha_-}^{\alpha^+} dp(p^2 - p_0(y)^2)) \tag{3.44}$$

$$\alpha_{\pm}\Pi_{,y} \pm \pi\tilde{\phi}(y) \tag{3.45}$$

with the stationary equation

$$p^2 - p_0(y)^2 = 0\,. \tag{3.46}$$

As a final comparison we mention that the representation obtained can be directly compared [37] to the string equation

$$[P, Q] = 1\,. \tag{3.47}$$

which is based on the KdV data

$$Q = d^2 - u \tag{3.48}$$
$$P = \left(Q^{\frac{2k-1}{2}}\right)_+\,. \tag{3.49}$$

For $k = 2$ one has

$$P = d^3 - \frac{3}{4}\{u, d\}, \tag{3.50}$$

and in the semiclassical tree approximation

$$P^2 = d^6 - 3ud^4 + \frac{9}{4}u^2 d^2\,. \tag{3.51}$$

Using $Q = d^2 - u$ one has

$$P^2 = Q^3 - \frac{3}{4}uQ + \frac{1}{4}u^3\,. \tag{3.52}$$

This is indeed identical to the stationary collective field equation established above. A precise correspondence reads

$$P \leftrightarrow \pi\tilde{\phi} \tag{3.53}$$
$$Q \leftrightarrow y \tag{3.54}$$

and $u = \Lambda(= t^{1/2})$ from the string equation.

4. Instanton solution in F-P field theory

In this section we discuss how the instanton solution appears in Fokker-Planck collective field theory. Towards this end, separate out an eigenvalue

$$\phi(x) = \tilde{\phi}(x) + \frac{1}{N}\delta(x - y)\,. \tag{4.1}$$

The field theory potential derived in the previous section splits into two contributions

$$\begin{aligned} V_1 &= \frac{1}{2N}\int dx\delta(x-y)\left(⨍\frac{\phi(x')}{x-x'}dx' - \frac{1}{2}v'(x)\right)^2 \\ &= \frac{1}{2N}\left(⨍\frac{\phi(x')}{y-x'}dx' - \frac{1}{2}v'(y)\right)^2, \end{aligned} \tag{4.2}$$

and

$$V_2 = \frac{1}{2}\int dx\tilde{\phi}(x)\left(\fint \frac{\tilde{\phi}(x')}{x-x'}dx' + \frac{1}{N(x-y)} - \frac{1}{2}v'(x)\right)^2$$
$$= \frac{1}{2}\int dx\left[\frac{\pi^2}{3}\tilde{\phi}^3(x) - \int \frac{1}{2}\tilde{\phi}(x')\tilde{\phi}(x)(1-g(x+x'))dx' + \frac{1}{4}(v'(x))^2\tilde{\phi}(x) - \right.$$
$$\left. -\frac{1}{N}\frac{\tilde{\phi}(x')v'(x)}{(x-y)} + \frac{2}{N}\frac{\tilde{\phi}(x)}{x-y}\fint \frac{\tilde{\phi}(x')}{x-x'}dx' + \frac{1}{N^2}\frac{\tilde{\phi}(x)}{(x-y)^2}\right]. \tag{4.3}$$

From the expression for the effective potential derived in the last section, it is easy to see it has a minimum value of zero. This minimum is achieved if y is chosen to satisfy

$$\fint \frac{\phi(x')}{y-x'}dx' = \frac{1}{2}v'(y)\,, \tag{4.4}$$

and we minimize V_2 with respect to $\tilde{\phi}(x)$. The equation of motion following from this minimization is

$$0 = \pi^2\tilde{\phi}^2 + \frac{1}{4}(v'(x))^2 - \frac{1}{N}\frac{v'(x)}{x-y} + \frac{1}{N^2(x-y)^2} - \int dx'\tilde{\phi}(x')(1-gx'-gx)$$
$$+ \frac{2}{N(x-y)}\int dx'\frac{\tilde{\phi}(x')}{x-x'} - \frac{2}{N}\int dx'\frac{\tilde{\phi}(x')}{(x'-y)(x-x')}. \tag{4.5}$$

This equation is that of a Fermi surface but now with a deformation induced by the single eigenvalue instanton. Various deformations have been considered in the literature, e. g. [15]. This equation can be easily solved perturbatively in $\frac{1}{N}$. The leading solution solves

$$0 = \pi^2\tilde{\phi}_0^2 + \frac{1}{4}(v'(x))^2 - \int dx'\tilde{\phi}_0(x')(1-gx'-gx). \tag{4.6}$$

Noting that $\int \tilde{\phi}_0(x)dx = 1 - \frac{1}{N}$, we have

$$0 = \pi^2\tilde{\phi}_0^2 + \frac{1}{4}(v'(x))^2 - (1-\frac{1}{N})(1-gx) + c\,, \tag{4.7}$$
$$c = g\int dx'\tilde{\phi}_0(x')x' \equiv c_0 + \frac{c_1}{N}\,. \tag{4.8}$$

The solution to first order in $\frac{1}{N}$ solves

$$0 = 2\pi^2\tilde{\phi}_0\tilde{\phi}_1 - \frac{v'(x)}{x-y} - \int dx'\tilde{\phi}_1(x')(1-gx'-gx) + \frac{2}{x-y}\int dx'\frac{\tilde{\phi}_0(x')}{y-x'}\,. \tag{4.9}$$

Noting that $\int \phi_1(x)dx = 0$, we have

$$0 = 2\pi^2\tilde{\phi}_0\tilde{\phi}_1 - \frac{v'(x)}{x-y} + \int dx'\tilde{\phi}_1(x')gx' + \frac{2}{x-y}\int dx'\frac{\tilde{\phi}_0(x')}{y-x'}\,. \tag{4.10}$$

It is now a simple task to obtain

$$
\begin{aligned}
\tilde{\phi}_1 &= \frac{1}{\pi}\frac{v'(x) - 2\int dx' \frac{\tilde{\phi}_0(x')}{y-x'}}{2\pi(x-y)\tilde{\phi}_0} - \frac{\int dx' \tilde{\phi}_1(x') g x'}{2\pi^2\tilde{\phi}_0} \\
&= \frac{1}{\pi}\frac{v'(y) - 2\int dx' \frac{\tilde{\phi}_0(x')}{y-x'}}{2\pi(x-y)\tilde{\phi}_0} - \frac{\int dx' \tilde{\phi}_1(x') g x'}{2\pi^2\tilde{\phi}_0} + \frac{1}{\pi}\frac{v'(x)-v'(y)}{2\pi(x-y)\tilde{\phi}_0}\,.
\end{aligned}
\tag{4.11}
$$

For any polynomial potential, $v'(x) - v'(y) \propto x - y$ so that the last term in $\phi_1(x)$ has no singularity at $x = y$. This result should be compared to the correction to the density obtained in section 2. We will be content to demonstrate agreement between the singular term when $x \to y$ in both expressions. The singular term corresponds to the isolated eigenvalue, i.e. to the instanton. The non-singular terms describe how the density of the other eigenvalues is distorted. The result for the singular term, from section 2, is

$$
\frac{1}{\pi}\frac{1}{y-x}\frac{\sqrt{y-a_0}\,\sqrt{y-b_0}}{\sqrt{a_0-x}\,\sqrt{x-b_0}}\,. \tag{4.12}
$$

The Fokker-Planck collective field theory gives

$$
\frac{1}{\pi}\frac{v'(y) - 2\int dx' \frac{\tilde{\phi}_0(x')}{y-x'}}{2\pi(x-y)\tilde{\phi}_0} = \frac{1}{\pi}\frac{1}{y-x}\frac{\sqrt{y-a_0}\sqrt{y-b_0}}{\sqrt{a_0-x}\sqrt{x-b_0}}\frac{f(y)}{f(x)}, \tag{4.13}
$$

for the same term. In this expression, $f(x)$ is a polynomial whose details are fixed by the potential. The agreement between the above two expressions follows upon noting that

$$
\begin{aligned}
\frac{1}{\pi}\frac{1}{y-x}\frac{\sqrt{y-a_0}\,\sqrt{y-b_0}}{\sqrt{a_0-x}\,\sqrt{x-b_0}}\frac{f(y)}{f(x)} &= \frac{1}{\pi}\frac{1}{y-x}\frac{\sqrt{y-a_0}\,\sqrt{y-b_0}}{\sqrt{a_0-x}\,\sqrt{x-b_0}} + \\
&\quad + \frac{1}{\pi}\frac{1}{y-x}\frac{\sqrt{y-a_0}\,\sqrt{y-b_0}}{\sqrt{a_0-x}\,\sqrt{x-b_0}}\frac{f(y)-f(x)}{f(x)}\,.
\end{aligned}
\tag{4.14}
$$

The second term on the right hand side is not singular as $x \to y$, so that the equality is established.

We will now consider the behaviour of $\tilde{\phi}_1$ in the double scaling limit. From section 2, we know that

$$
\tilde{\phi}_1 = -\frac{1}{\pi}\frac{1}{y-x}\frac{\sqrt{y-a_0}\,\sqrt{y-b_0}}{\sqrt{a_0-x}\,\sqrt{x-b_0}} + \frac{1}{\pi}\frac{1}{\sqrt{a_0-x}\sqrt{x-b_0}}\,. \tag{4.15}
$$

Taking the double scaling limit as

$$
g = g_*\left(1 - \frac{3^{1/2}}{16}\gamma^2 a^2 t\right), \tag{4.16}
$$

$$
x = x_* - \gamma a z\,, \tag{4.17}
$$

$$
y = x_* + \gamma a w\,, \tag{4.18}
$$

we obtain

$$
\rho_1 = -\frac{1}{\pi\gamma}\frac{a^{3/2}}{N a^{5/2}}\frac{1}{w+z}\frac{\sqrt{w+\sqrt{t}}}{\sqrt{z-\sqrt{t}}}\,. \tag{4.19}
$$

Note that this is exactly of the form expected for a g_s effect and that the term which is regular as $y \to x$ does not survive in this limit. This expression is again universal, and independent of criticality, once the appropriate renormalized string coupling is identified. It is in agreement with expressions obtained in conformal backgrounds ref [18].

Consequently we have seen in this section that the instanton solution can be generated in the (first) nonlocal form of the collective field equation. Once the eigenvalue is separated a transition to the local form can be made. It takes the form of a deformed Fermi surface with a deformation induced by the eigenvalue. At the original level of the nonlocal equation however one has background independence, the instanton and the vacuum are both solutions of one and the same equation.

5. Conclusions

In this work we addressed the question whether D-instanton solutions can be obtained as solutions of a closed string field theory. Due to the very special (stringy) nature of these objects, reflected in the fact that their action is given by $1/g$ as opposed to the standard $1/g^2$ of field theory, one could indeed expect that this is not possible. The studies of ref [17] where the instanton effects were demonstrated at the level of matrices also pointed out some difficulties in obtaining these effects through loop or string equations. We believe that the instanton effects can be recovered in the continuum collective field theory. This required exploiting some subtle and nontrivial features of this field theory. We now list what these features were. First, in evaluation of the free energy (or any correlator) one has to consider the presence of a nontrivial jacobian which is seen to supply the crucial factor of N toward the weight of an instanton contribution. Then at the level of field equations we emphasized the existence of two formally equivalent equations. The first form was nonlocal while the second took the local form of a Fermi sea. It is in the first nonlocal version of the equation that the separation of a single eigenvalue D-instanton is possible. Consequently one can state that the vacuum and the instanton are two (different) solutions of one and the same equation. Considering the second local form of the equation one sees a deformation (induced by the instanton). The collective field theory equations that we considered involve more degrees of freedom then the loop equations. In the sense of analytic continuation they involve both positively and negatively dressed loops. We believe that this difference is responsible for the fact that (instanton) solutions are present in collective but not in loop equations. Furthermore one also had the effect of the jacobian which did not contribute to the form of the solution (in leading order) but did contribute to the overall weight of the instanton contribution. It is also likely that the D-instantons can also be described (probably more elegantly) in the framework of extended open-closed field theory. After all, the eigenvalue deformations can be represented in terms of a quark integral. As a step toward this possible description of instantons we note that the nontrivial jacobian that we emphasized very likely also posses an interpretation in terms of open string degrees of freedom.

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A. BIPZ Solution for the eigenvalue density

In this appendix we summarize the solution to the integral equation

$$\fint \frac{\phi(x')}{x-x'}dx' = \frac{1}{2}\left(x - gx^2 + \frac{2}{N(y-x)}\right). \tag{A.1}$$

Equations of this type have been investigated in connection with matrix theories of Penner type [38] and also theories of open and closed strings [39, 40]. The equation is solved by extension of well known technique [27]. For the case of a cubic potential this was given in [41]. One begins by defining

$$G(z) = \fint \frac{\phi(x')}{z-x'}dx', \tag{A.2}$$

where z is a complex number. From the properties of the principal value prescription, it follows that

$$G(x \pm i\epsilon) = \frac{1}{2}\left(x - gx^2 + \frac{2}{N(y-x)}\right) \mp i\pi\phi(x) \tag{A.3}$$

where x lies in the support of the density $\phi(x)$. Analytic structure of $G(z)$ suggests the ansatz

$$G(z) = -g\frac{z^2}{2} + \frac{z}{2} - \frac{1}{N(z-y)} + \left(\bar{a}z + \bar{b} + \frac{F}{y-z}\right)\sqrt{z-a}\sqrt{z-b}. \tag{A.4}$$

The parameters $\bar{a}$, $\bar{b}$, F, a and b are determined by requiring that (i) $G(z)$ does not have a pole at $z = y$ and (ii) that as $z \to \infty$, $G(z) \to \frac{1}{z}$. The solution is

$$\phi(x) = -\frac{1}{\pi}\left(\frac{g}{2}x - \frac{1}{2} + \frac{g}{4}(b+a) - \frac{1}{N\sqrt{y-a}\sqrt{y-b}}\frac{1}{y-x}\right)\sqrt{a-x}\sqrt{x-b} \tag{A.5}$$

with a and b solutions to the equations (in what follows $s = a + b$ and $d = a - b$)

$$\frac{-16}{N\sqrt{y-a}\sqrt{y-b}} + s(2gs-4) + gd^2 = 0, \tag{A.6}$$

$$-(\frac{s}{2} - y)\frac{1}{N\sqrt{y-a}\sqrt{y-b}} + \frac{d^2}{16}(1-gs) = 1. \tag{A.7}$$

We would like to compare this to the result of section 2. Towards this end expand the density perturbatively in $\frac{1}{N}$. To capture the next to leading order, set

$$a = a_0 + \frac{1}{N}a_1 \qquad b = b_0 + \frac{1}{N}b_1 \qquad s = s_0 + \frac{1}{N}s_1 \qquad d = d_0 + \frac{1}{N}d_1, \tag{A.8}$$

and

$$\phi(x) = \phi_0 + \frac{1}{N}\phi_1 \,. \tag{A.9}$$

We find

$$\phi_0(x) = \frac{1}{2\pi}\left(1 - gx - \frac{g}{2}(b_0 + a_0)\right)\sqrt{a_0 - x}\sqrt{x - b_0}\,, \tag{A.10}$$

$$s_0(2gs_0 - 4) + gd_0^2 = 0\,, \qquad \frac{d_0^2}{16}(1 - gs_0) = 1\,, \tag{A.11}$$

for the leading order and

$$\begin{aligned}
\phi_1(x) = \frac{1}{2\pi}\sqrt{a_0 - x}\sqrt{x - b_0}\Bigg[&\frac{\sqrt{y - a_0}\sqrt{y - b_0}}{(a_0 - b_0)}\left(\frac{2}{(y - b_0)(x - b_0)} - \frac{2}{(y - a_0)(x - a_0)}\right) - \\
&- \frac{gs_1}{2} + \left(1 - gx - \frac{g}{2}(a_0 + b_0)\right)\left(\frac{a_1}{2(a_0 - x)} + \frac{b_1}{2(b_0 - x)}\right)\Bigg] - \\
&- \frac{1}{\pi}\frac{\sqrt{y - a_0}\sqrt{y - b_0}}{\sqrt{a_0 - x}\sqrt{x - b_0}(y - x)}
\end{aligned} \tag{A.12}$$

$$-\frac{16}{\sqrt{y - a_0}\sqrt{y - b_0}} + 4s_1(gs_0 - 1) + 2gd_0d_1 = 0\,, \tag{A.13}$$

$$-\left(\frac{s_0}{2} - y\right)\frac{1}{\sqrt{y - a_0}\sqrt{y - b_0}} + \frac{d_0d_1}{8}(1 - gs_0) - g\frac{d_0^2}{16}s_1 = 0\,, \tag{A.14}$$

for the subleading order. The coefficient of the $\frac{1}{y-x}$ pole is in perfect agreement with the result of section 2.

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